In the undeformed state with complete LRO, the NNN $\langle 100\rangle$ directions contain 2AA+2BB pairs in the two unit cells of Fig. 1(a). After (110) $[\bar{1}11]$ slip, Fig. 1(b), the [001] direction which lies on the slip plane, is undisturbed. The [100] and [010] directions, on the other hand, now contain four AB pairs each. Hence the slip has resulted in a (negative) gain of -2 BB pairs. For partial LRO, $\Delta N_{\mathrm{BB}} = -2s^2$ (see Appendix), or $-s^2/a^2\sqrt{2}$ per (110) area. Multiplying by the appropriate factors as in eqns. (2) and (3), we obtain, per unit volume,

$$\Delta N_{\mathrm{BB}} = -\tfrac{1}{4} N p_0 p' s^2 S \qquad (8)$$

in both [001] and [010] directions. Thus, from eqn. (1),

$$\begin{aligned} E &= -\tfrac{1}{4} N l_2 p_0 p' s^2 S[(1\cdot\alpha_1 + 0\cdot\alpha_2 + 0\cdot\alpha_3)^2 \\ &\quad + (0\cdot\alpha_1 + 1\cdot\alpha_2 + 0\cdot\alpha_3)^2] \\ &= -\tfrac{1}{4} N l_2 p_0 p' s^2 S(\alpha_1^2 + \alpha_2^2) \\ &= \tfrac{1}{4} N l_2 p_0 p' s^2 S\alpha_3^2 , \end{aligned} \qquad (9)$$

where l_2 refers to the value of l for NNN pairs. The functional dependence of eqn. (9) indicates that the [001] direction, which lies on the slip plane (110), is the effective BB pair direction. If the value of l_2 is assumed negative, by analogy with l, [001] is an easy axis of magnetization. In the general case, we have

$$E = \tfrac{1}{4} N l_2 p_0 p' s^2 \sum_i |S_i| (\delta_{1i}^2 \alpha_1^2 + \delta_{2i}^2 \alpha_2^2 + \delta_{3i}^2 \alpha_3^2) , \qquad (10)$$

where δ_{1i}, δ_{2i}, δ_{3i} are the direction cosines of the $\langle 100\rangle$ direction lying on the ith $\{110\}$ slip plane.

For the case of short-range order, the (negative) gain in BB pairs along [100] or [010] is $-\sigma_2/2\sqrt{2}a^2$ per unit (110) slipped area (see Appendix), where σ_2 is the Bethe SRO parameter for NNN pairs. After inserting the appropriate factors as in eqn. (6) and combining the direction cosines as in eqn. (10), the expression of E for the SRO case becomes

$$E = \tfrac{1}{4} N l_2 p' \sigma_2 \sum_i |S_i| (\delta_{1i}^2 \alpha_1^2 + \delta_{2i}^2 \alpha_2^2 + \delta_{3i}^2 \alpha_3^2) . \qquad (11)$$

The combined results of eqns. (10) and (11) then lead to a slip-induced anisotropy energy of

$$E_{\mathrm{NNN}} = \tfrac{1}{4} E_2 \sum_i |S_i| (\delta_{1i}^2 \alpha_1^2 + \delta_{2i}^2 \alpha_2^2 + \delta_{3i}^2 \alpha_3^2) , \qquad (12)$$

where $E_2 \equiv N l_2 p' (p_0 s^2 + \sigma_2)$, for the next nearest-neighbor case.

(c) *Applications to rolling*

Equations (7) and (12) have been applied to calculate the slip-induced anisotropy obtained by rolling single crystals. As Fig. 2 shows, the rolled texture of an alloy near the 50% Fe–50% Co composition may be considered as a band of orientations $\{001\}$ to $\{111\}\langle\bar{1}10\rangle$, that is, the rolling direction is a $\langle\bar{1}10\rangle$ orientation, but the rolling plane consists of a continuous rotation about $\langle\bar{1}10\rangle$, from $\{001\}$ to $\{111\}$ positions. Hence calculations were made for $(001)[\bar{1}10]$, $(115)[\bar{1}10]$, $(112)[\bar{1}10]$, and $(111)[\bar{1}10]$ orientations which comprise the texture spread. (For completeness, the $(110)[\bar{1}10]$ orientation was also analyzed.) It may be added that the rolled texture of Fig. 2 is common to most b.c.c. alloys.

The procedure in the calculations is essentially identical with that adopted previously for $FeNi_3$[4]. In Table I are listed the values of the macroscopic strain components ε_{xx}, ε_{yz} etc. in terms of the slip

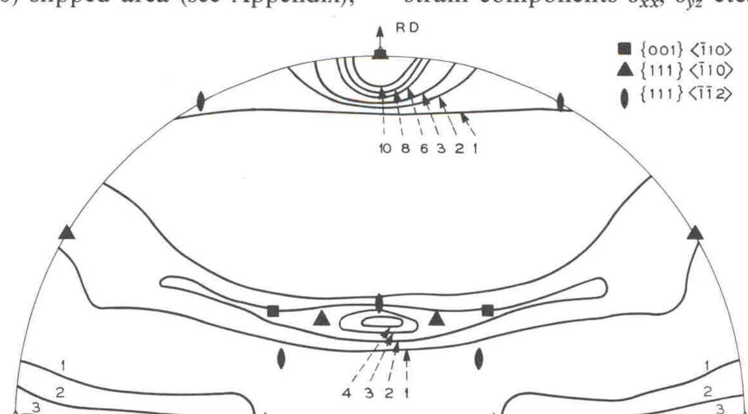

Fig. 2. $\{110\}$ pole figure of cold-rolled polycrystalline Remendur (49% Fe–49% Co–4% V) after 95% thickness reduction. Ideal texture can be described as $\{001\}$ to $\{111\}$ $\langle\bar{1}10\rangle$. (A.T. English and G.Y. Chin, unpublished.)

TABLE I: VALUES OF ε, n AND δ (REFERRED TO CUBIC AXES) FOR THE TWELVE $\{110\}\langle 111\rangle$ SLIP SYSTEMS

No. of slip system	*Slip plane*	*Slip direction*	$2\varepsilon_{xx}$	$2\varepsilon_{yy}$	$2\varepsilon_{zz}$	$4\varepsilon_{yz}$	$4\varepsilon_{zx}$	$4\varepsilon_{xy}$	$2n_1n_2$	$2n_2n_3$	$2n_3n_1$	δ_1^2	δ_2^2	δ_3^2
1	$(01\bar{1})$	$[111]$	0	S_1	$-S_1$	0	$-S_1$	S_1	0	-1	0	1	0	0
2	$(10\bar{1})$	$[111]$	S_2	0	$-S_2$	$-S_2$	0	S_2	0	0	-1	0	1	0
3	$(1\bar{1}0)$	$[111]$	S_3	$-S_3$	0	$-S_3$	S_3	0	-1	0	0	0	0	1
4	(101)	$[11\bar{1}]$	S_4	0	$-S_4$	S_4	0	S_4	0	0	1	0	1	0
5	(011)	$[11\bar{1}]$	0	S_5	$-S_5$	0	S_5	S_5	0	1	0	1	0	0
6	$(1\bar{1}0)$	$[11\bar{1}]$	S_6	$-S_6$	0	S_6	$-S_6$	0	-1	0	0	0	0	1
7	(110)	$[1\bar{1}1]$	S_7	$-S_7$	0	S_7	S_7	0	1	0	0	0	0	1
8	$(10\bar{1})$	$[1\bar{1}1]$	S_8	0	$-S_8$	S_8	0	$-S_8$	0	0	-1	0	1	0
9	(011)	$[1\bar{1}1]$	0	$-S_9$	S_9	0	S_9	S_9	0	1	0	1	0	0
10	$(01\bar{1})$	$[\bar{1}11]$	0	S_{10}	$-S_{10}$	0	S_{10}	$-S_{10}$	0	-1	0	1	0	0
11	(101)	$[\bar{1}11]$	$-S_{11}$	0	S_{11}	S_{11}	0	S_{11}	0	0	1	0	1	0
12	(110)	$[\bar{1}11]$	$-S_{12}$	S_{12}	0	S_{12}	S_{12}	0	1	0	0	0	0	1

TABLE II: SUMMARY OF RESULTS BASED ON $\{110\}\langle 111\rangle$ SLIP

Rolling plane	*Rolling direction*	*Active slip systems*	$\lvert S_i\rvert$	E_{NN}	*Easy* axis*	E_{NNN}	*Easy* axis*
(001)	$[\bar{1}10]$	8,9,10,11	$r/2$	0	—	$-(\frac{1}{4}E_2 r)\alpha_3^2$	RD+TD
(115)	$[\bar{1}10]$	4,5,8,9,10,11	$\lvert S_4\rvert=\lvert S_5\rvert=2r/27$ $\lvert S_8\rvert=\lvert S_{10}\rvert=r/6$ $\lvert S_9\rvert=\lvert S_{11}\rvert=5r/6$	$(\frac{20}{81}E_1 r)[\alpha_3(\alpha_1+\alpha_2)]$	RPN	$-(\frac{29}{108}E_2 r)\alpha_3^2$	RD
(112)	$[\bar{1}10]$	4,5,9,11	$\lvert S_4\rvert=\lvert S_5\rvert=r/3$ $\lvert S_9\rvert=\lvert S_{11}\rvert=r$	$(\frac{4}{9}E_1 r)[\alpha_3(\alpha_1+\alpha_2)]$	RPN	$-(\frac{1}{3}E_2 r)\alpha_3^2$	RD
(111)	$[\bar{1}10]$	1,2,4,5,9,11	$\lvert S_1\rvert=\lvert S_2\rvert=r/6$ $\lvert S_4\rvert=\lvert S_5\rvert=r/2$ $\lvert S_9\rvert=\lvert S_{11}\rvert=r$	$(\frac{4}{9}E_1 r)[\alpha_3(\alpha_1+\alpha_2)]$	RPN	$-(\frac{5}{12}E_2 r)\alpha_3^2$	RD
(110)	$[\bar{1}10]$	1,2,4,5,8,9,10,11	$r/2$	0	—	$-(\frac{1}{2}E_2 r)\alpha_3^2$	RD+RPN

* Relative among the three symmetry directions of rolled strip.
RP-rolling plane, RD-rolling direction, RPN-rolling plane normal.

density $|S_i|$ for the twelve $\{110\}\langle 111\rangle$ slip systems. These values are referred to cubic axes of the crystal and were computed from the equation

$$\varepsilon_{ij} = \frac{\gamma}{2}(n_i d_j + n_j d_i) \qquad i,j = x,y,z$$

$$= \frac{\sqrt{6}}{4} S(n_i d_j + n_j d_i)\,, \qquad (13)$$

where γ is the glide-shear, and n_i and d_i are the direction cosines of the slip plane normal and the slip direction, respectively. (See Appendix, ref. 4.) Values of (n_{1i}, n_{2i}, n_{3i}) and $(\delta_i, \delta_{2i}, \delta_{3i})$ for use in eqns. (7) and (12) are also included in Table I.

For a given crystal orientation, the method of Bishop and Hill[13,14] is used as a first step to determine which of the 12 possible $\{110\}\langle 111\rangle$ slip systems must operate to accommodate the (rolling) deformation. Next, the macroscopic strain components referred to specimen axes* are converted to those referred to cubic axes by the appropriate coordinate transformation. Table I can then be used to relate the strain components (now referred to cubic axes) and the slip density S_i of the active slip systems which have been determined by the Bishop and Hill method. Finally, the values of S_i for use in eqns. (7) and (12) are solved in terms of the strain components.

It may be noted that for $\{110\}\langle 111\rangle$ slip in b.c.c. alloys, the slip planes and slip directions are merely interchanged from those of $\{111\}\langle 110\rangle$ slip

* If 1, 2, 3 refer to the rolling plane normal, transverse direction, and rolling direction of the specimen respectively, the strain components during rolling are given by $\varepsilon_{11} = -\varepsilon_{33}$, $\varepsilon_{22} = 0$, $\varepsilon_{23} = \varepsilon_{31} = \varepsilon_{12} = 0$.